

Calculation of the Current Density in the Contacts of a Thin Film Resistor

Abstract: The two-dimensional boundary value problem appropriate to current flow in a film resistor is examined. A simple closed-form solution for current density into the contact is found to exist for the important case of a thin film resistor with extended lands. The spatial dependence of the current density into the contact is found to be similar to that obtained by Kennedy and Murley for the diffused resistor, with film thickness entering the functional dependence in a role analogous to the diffusion length of the dopant ion in the diffused resistor.

Introduction

The trend toward higher current densities in integrated circuit components, and the importance of current density in promoting electrical degradation, have given impetus to the mathematical analysis of certain flow problems that have been neglected in the past. A recent publication by Kennedy and Murley¹ gives a numerical solution to the problem of the current distribution in the contacts of a diffused resistor. The treatment is general in that it first gives a solution for the diffusion problem involved in the fabrication of the resistor, followed by a solution of the two-dimensional flow problem using the calculated conductivity profile. The problem considered in the present communication is the constant-conductivity situation appropriate to a film resistor. A solution for the current density exists in closed form, and it is shown that in the important case of the "extended land," such as is used in cermet and nichrome resistors, a surprisingly simple solution exists for the current density and its integral. It is further found that the spatial dependence of current density in the contact closely resembles that obtained by Kennedy and Murley for the diffused resistor, with the thickness of the film playing a role analogous to that of the diffusion length of the dopant ion in the diffused resistor calculation.

The calculation

Consider a resistor of uniform electrical conductivity σ and thickness t , situated in the complex $\zeta = \xi + j\eta$ plane shown in Fig. 1(a), and having infinite extent in the

positive and negative ξ directions. Contacts of infinite conductivity and finite length L are applied as shown, and it is desired to solve the boundary value problem for $J(x)$, the current density into the contact through the plane $\eta = 0$. The origin in the ζ system is at $0'$, and the origin for the coordinate x is at 0 , which is the inner edge of the contact along the positive ξ axis. The resistor and its lands have finite width W in the direction perpendicular to the plane of the figure. Since the corresponding two-dimensional boundary value problem has been considered in the literature,² we shall first outline the solution and then apply suitable simplifications.

The strip in the ζ plane is mapped into the upper half of the z_1 plane, Fig. 1(b), by the transformation

$$\zeta = \frac{t}{\pi} \ln \left[\left(z_1 - \frac{\pi}{2t} \right) / \left(z_1 + \frac{\pi}{2t} \right) \right] - jt, \quad (1)$$

where the constants have been chosen so that the origin in the ζ plane is transformed into the origin in the z_1 plane, and the point $\zeta = -jt$ is transformed into infinity in the z_1 plane. The equipotentials between $\xi = d/2$ and $\xi = (d/2) + L$, $\eta = 0$, are transformed into equipotentials between points α and β in the z_1 plane by the substitutions

$$\alpha = \frac{\pi}{2t} \tanh \frac{\pi d}{4t} \quad \text{and} \quad (2)$$

$$\beta = \frac{\pi}{2t} \tanh \frac{\pi}{2t} \left(\frac{d}{2} + L \right),$$

the image equipotentials along the negative ξ axis being the negative of Eq. (2).

The author is located at the IBM Components Division Laboratory, E. Fishkill, New York 12533.

The derivative of the function w mapping the inside area of the rectangle shown in Fig. 1(c) onto the upper half of the z_1 plane is given by the Schwarz-Christoffel transformation

$$\frac{dw}{dz_1} = (z_1^2 - \alpha^2)^{-1/2} (z_1^2 - \beta^2)^{-1/2}. \quad (3)$$

The relative current density³ (J_{rel}) may now be calculated to be

$$\begin{aligned} J_{rel} &= \left| \frac{dw}{d\xi} \right| = \left| \frac{dw}{dz_1} \frac{dz_1}{d\xi} \right| \\ &= \left| \left(\operatorname{sech}^2 \frac{\pi \xi}{2t} \right) \left\{ \left(\tanh^2 \frac{\pi \xi}{2t} - \tanh^2 \frac{\pi d}{4t} \right) \right. \right. \\ &\quad \times \left. \left[\tanh^2 \frac{\pi \xi}{2t} - \tanh^2 \frac{\pi}{2t} \left(\frac{d}{2} + L \right) \right] \right\}^{-1/2} \right|. \quad (4) \end{aligned}$$

• Calculating J_{rel} for a thin film resistor

Film resistors used in microelectronics frequently employ an "extended land," in which a conductive land completely overlays the resistive film except in those areas in which a resistor is to be fabricated, Fig. (2). This corresponds, in the present model, to making $L \gg t$. For this situation, $\tanh^2 (\pi/2t)[(d/2) + L] \rightarrow 1$, and Eq. (4) becomes

$$J_{rel} = \left| \left[\frac{1 - \tanh^2 (\pi \xi / 2t)}{\tanh^2 (\pi \xi / 2t) - \tanh^2 (\pi d / 4t)} \right] \right|^{1/2}. \quad (5)$$

The difference of the squares of the hyperbolic tangents may be written as

$$\begin{aligned} \tanh^2 (\pi \xi / 2t) - \tanh^2 (\pi d / 4t) \\ &= [\tanh (\pi \xi / 2t) + \tanh (\pi d / 4t)] \\ &\quad \times [\tanh (\pi \xi / 2t) - \tanh (\pi d / 4t)]. \quad (6) \end{aligned}$$

By using

$$\tanh a \pm \tanh b = \frac{\sinh (a \pm b)}{\cosh a \cosh b},$$

J_{rel} can be further reduced to

$$J_{rel} = \frac{\cosh (\pi d / 4t)}{\left[\sinh \frac{\pi}{2} \left(\frac{d}{t} + \frac{x}{t} \right) \sinh \frac{\pi x}{2t} \right]^{1/2}}. \quad (7)$$

In deriving Eq. (7) the coordinate x has been introduced, and we have set $\eta = 0$ preparatory to calculating the current density into the land.

In a thin film resistor the minimum distance between contacts is about 1 mil (25 μ m) and the maximum thickness is about 1 μ m, giving $d/2t$ a minimum value of approximately 12. The range of interest of Eq. (7) is thus $d/t \gg 1$.

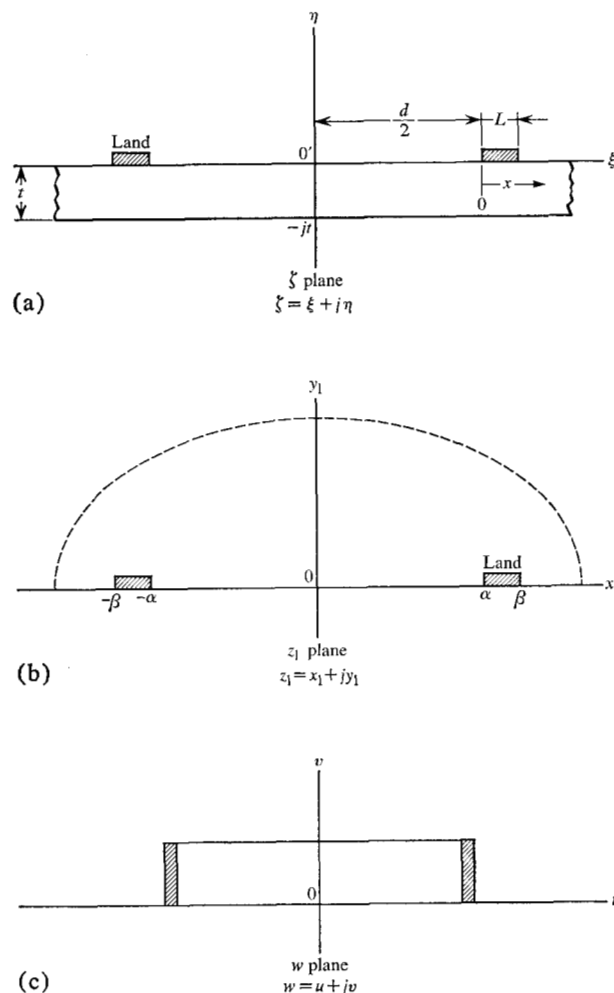
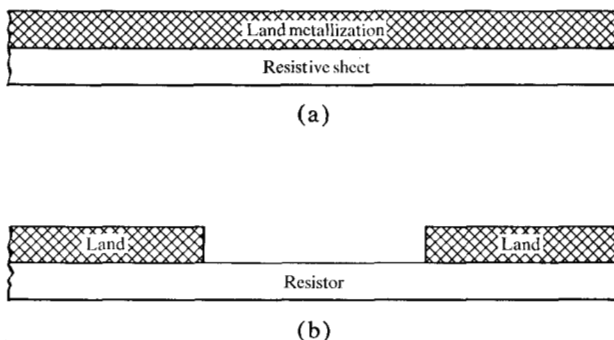


Figure 1(a) Geometry of the resistor in the complex ξ plane. (b) Geometry of Fig. 1(a) transformed into the complex z_1 plane. (c) Model solution of the boundary value problem (to be mapped onto the z_1 plane).

Figure 2 Cross section of typical thin film resistor, illustrating an "extended land." (a) Blanket deposition of land metal on resistive sheet. (b) Removal of shunt conductor to form resistor.



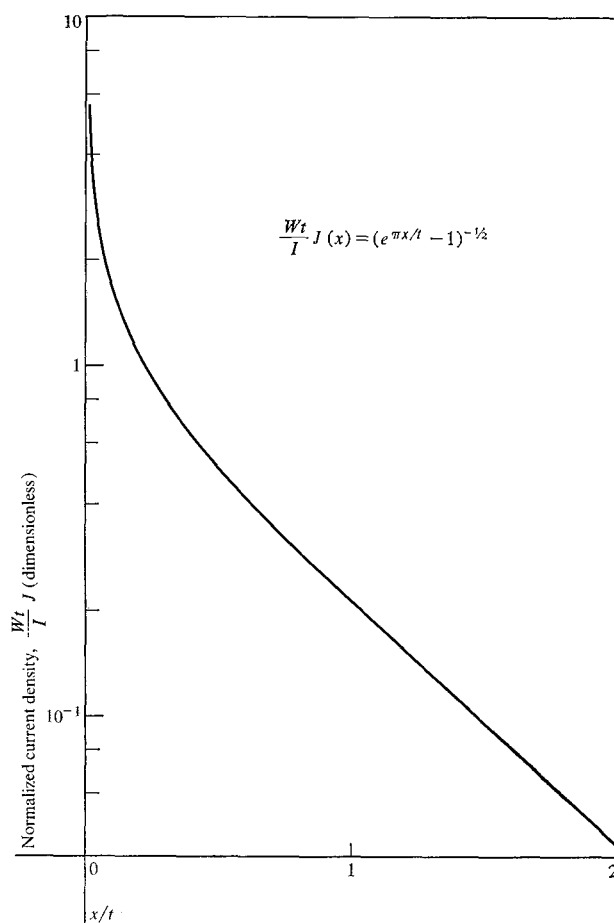


Figure 3 Current density vs. distance. (Distance normalized with respect to t .)

Evaluating Eq. (7) for d/t large gives:

$$\lim_{d/t \rightarrow \infty} J_{rel} = 1/(e^{\pi x/t} - 1)^{1/2}.$$

Normalizing, we have

$$\int_0^\infty J_{rel} dx = \left[\frac{2t}{\pi} \sec^{-1}(e^{\pi x/2t}) \right]_0^\infty = t.$$

The final result for current density is then

$$J(x) = \frac{I}{Wt} \frac{1}{(e^{\pi x/t} - 1)^{1/2}}, \quad (8)$$

where W is the width of the land (assumed equal to the width of the resistor) and I is the total current. The cumulative current flowing through the interface between contact and resistor is therefore

$$I_c(x) = W \int_0^x J(x) dx = \frac{2I}{\pi} \sec^{-1}(e^{\pi x/2t}). \quad (9)$$

Discussion

Equation (8) gives the current density $J(x)$ into the land as a function of distance from the inner edge of the con-

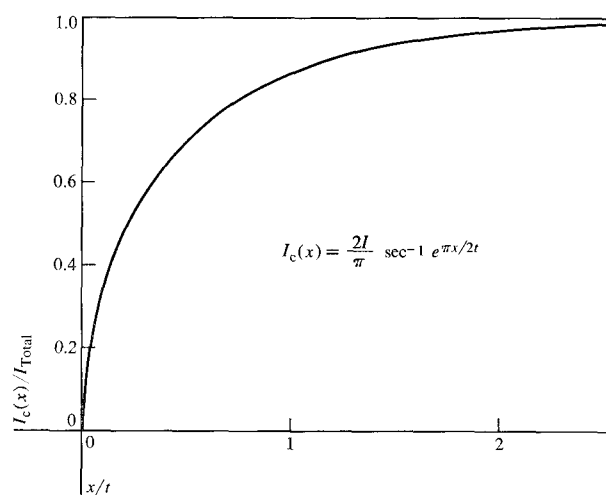


Figure 4 Integrated current vs. distance. (Distance normalized with respect to t .)

tact, and Eq. (9) gives the total current flowing from 0 to x . Current density is plotted as a function of distance in Fig. 3 and the integrated current is shown in Fig. 4. From Fig. 3, the current density at $x = 0.2t$ is approximately equal to the value expected in the resistive sheet in the approximation of uniform flow; that is, $J = I/Wt$. At $x = t$, J has about 21% of this value, while at $x = 0$ the current density becomes infinite. From the integral function, Fig. 4, approximately 87% of the total current into the contact flows in the region between $x = 0$ and $x = t$, while 97% is contained within $x = 2t$.

The problem considered here is analogous to the Kennedy-Murley diffused-resistor calculation, with the assumption of constant electrical conductivity in the resistive film and with boundaries delineated by the thin film geometry. Figure 5 compares the present result for $J(x)$ with the Kennedy-Murley relaxation solutions for n- and p-type instantaneous-source and constant-surface impurity concentration diffusions, while Figure 6 compares the current integrals $I_c(x)$. It is seen that the functional forms of the solutions are remarkably similar when one thinks of the thickness t of the thin film resistor as analogous to the diffusion length L_D of the dopant ion of the diffused resistor.

It is also of interest to obtain some estimate of the importance of the assumption of infinite conductivity in the lands. Whereas common resistive films may have a sheet resistivity as low as 5 ohms per square, if we choose for the sheet resistivity of the lands the high value of 50 milliohms per square, the conductance ratio of 100 suggests that the present solution should be a close approximation. Unpublished calculations by Kennedy and Murley provide an interesting estimate of the deviations to be expected for smaller conductance ratios. The numerical solution graphed in Fig. 7 was obtained by relaxation

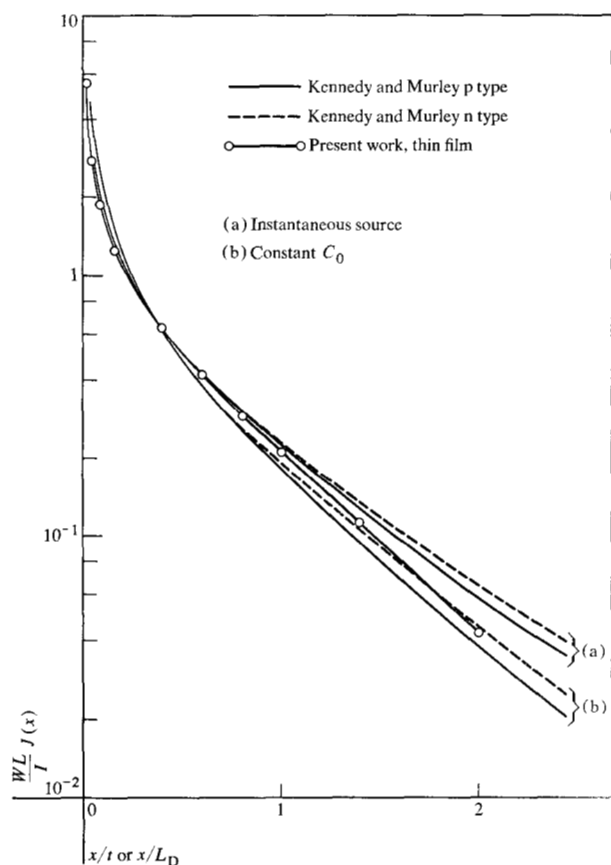


Figure 5 Comparison of the current density $J(x)$ for a thin film resistor with that for a diffused resistor.

methods for two sheets of equal thickness with a conductivity ratio of 2, as diagrammed in the figure. The solution for $J(x)$ given by Eq. (8) is also plotted and is seen to agree with the numerical solution within 25% for $0 < x < t$, an interval which includes 87% of the current. It thus appears that the assumption of infinite conductivity should not prove very restrictive in many situations of technical importance.

Acknowledgments

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References

1. D. P. Kennedy and P. C. Murley, *IBM J. Res. Develop.* **12**, 242 (1968).
2. "Integrated Silicon Device Technology," Vol. 1, *Research Triangle Institute Report ASD-TDR-63-316* (June, 1963).
3. See, for example, W. R. Smythe, *Static and Dynamic Electricity*, McGraw-Hill Publishing Co. Inc., New York 1950, p. 73.

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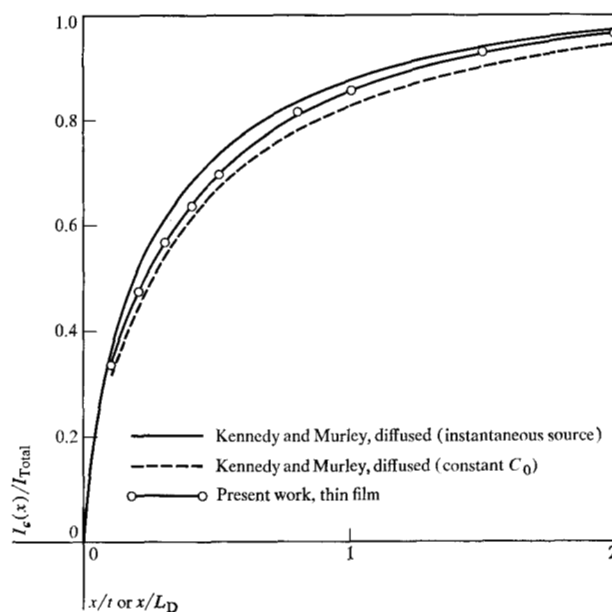


Figure 6 Comparison of integrated current $I_e(x)$ for a thin film with that for a diffused resistor.

Figure 7 Current density profiles compared. (a) Relaxation calculation for the geometry shown in the inset. (b) Result from Eq. (8), for $\sigma_{land} \rightarrow \infty$.

